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Interaction of two domain walls during spin-torque-induced coherent motion

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Abstract
We show that the application of a spin-polarized current to a double $\pi$ domain wall system with a variable distance between the walls results in an interaction between the two domain walls. The transmission spectrum changes from that of a spin-dependent resonant double barrier to one like a $2\pi$ wall. In addition, the spin torque on each individual wall creates coupled motion in the domain walls. The walls move independently with a fast speed at large separations, but slow considerably at small separations.

Keywords: domain wall motion, spin transfer torque, spin-dependent transport, multiple domain walls

(Some figures may appear in colour only in the online journal)

1. Introduction

The effects of spin transport in inhomogeneous magnetic systems have important implications for both the understanding of fundamental physics and the development of potential applications. Electrical generation of spin torque, which is a direct manifestation of the conservation of the angular momentum associated with spin, permits fast, localized electrical switching of magnetic domains [1–4], electrical driving of ferromagnetic resonance [5–9], and controlled generation of coherent magnons [10, 11]. The study of the motion of domain walls induced by this spin torque [12–31] may lead to novel spin torque devices [32–38]. Having multiple domain walls can lead to the formation of multiple rotation walls [39–42]. The treatment of multiple domain walls in a single system[43–46] is a necessary next step, as many of these future devices will require the manipulation of more than one domain wall at a time.

Here we calculate spin transport properties and spin torque for a pair of $\pi$ walls in a ferromagnetic material separated by a domain of variable size, with the central domain’s magnetization oriented 180 degrees from the magnetization orientation of the regions on the far left and far right, to which electrical contact is made. A Stoner electronic structure model is used, corresponding to a free-electron dispersion, a uniform spin splitting and no spin–orbit interaction. This simplified electronic structure model for the ferromagnet illuminates the qualitative behavior of nearby domain walls, unencumbered by the material details of a specific choice for the ferromagnet. The carrier density is chosen so that the Fermi energy is less than the spin splitting energy, so the spin polarization is 100\% at $T = 0$ K. For Fermi energies above the spin splitting energy the partial spin polarization will partially compensate the spin torque produced, reducing the domain wall velocities and interactions correspondingly. Thus we focus on the nearly 100\% spin-polarized case, only reduced by thermally activated minority carriers. Using a model Hamiltonian [47] for the coherent transport of spin-polarized carriers through a domain wall in the absence of spin–orbit interaction, and a piecewise linear approximation [41] for the rotation of magnetization inside the domain walls, we calculate, using a transfer-matrix approach, the energy-resolved transmission and reflection coefficients, the energy-resolved spin torque, and the total spin torque as a function of separation for each domain wall individually and for the system as a whole.

We find that the spin torque in each domain wall, and thus the velocity of each domain wall, has a unique non-trivial dependence on the separation of the walls. The walls move independently at the same speed when the walls are far apart, but slower when the walls are closer together. This suggests
a limitation on how close together the domain walls can be placed on a ‘racetrack’ [33], ultimately putting a lower limit on the size of the bits.

Although there are effects that have been neglected, including nonparabolic electronic dispersion, spin relaxation, and spin–orbit interaction, which will modify the quantitative results, the general features of the results we obtain should apply to all ferromagnets. In order to focus on the effects associated with spin torque we also neglect dipolar interactions between the domains, which is justified for thin domain walls or for materials with low saturation magnetization. For the same reason, we also perform these calculations using domain walls of a strictly finite size. For more realistic domain walls, in which the gradient of the magnetization falls off with distance more smoothly, instead of abruptly vanishing at the edge of a specific domain wall thickness, one may also expect exchange-mediated motion of domain walls towards or away from each other.

2. Electronic structure theory of the two domain walls

A schematic of the two-wall system is shown in figure 1(a). Two domain walls are separated by a domain, the magnetization of which is antiparallel to the magnetization at the far right and far left; these outer regions are connected to the electrical leads. As shown, these are Néel walls, energetically favorable in thin films [48, 49]. We use a simple model for the domain walls that yields analytic results for the domain wall wavefunctions. The key approximations, as in [41], are (1) the magnetic material is assumed to be a parabolic-band Stoner model; the spin splitting between spin-up and spin-down is assumed uniform, (2) spin–orbit effects in the electronic structure are neglected, and (3) the domain wall is assumed to lie between two regions with a uniform exchange field, and the domain wall itself has a piecewise constant variation of the exchange field with position. As variations in the band structure and spin–orbit effect can be vary wide from material to material, when these are important our model will provide only qualitative results.

The domain walls are oriented such that they represent a full 360 degree rotation in magnetization, rather than 180 degrees in opposite directions. Spin polarized carriers are injected from the left side. The exchange field for such a system can be approximated as:

\[ \mathbf{B} = B_0 \left[ \sin \theta(x) \mathbf{i} + \cos \theta(x) \mathbf{j} \right], \]

where the form of \( \theta(x) \) is shown in figure 1(b) for a pair of 1.8 nm thick domain walls separated by 5 nm, and is based on a realistic form for the magnetization inside a domain wall [41, 50].

The Schrödinger equation for the system with this exchange field is [47, 51]

\[
\begin{align*}
\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} - \frac{\Delta}{2} \begin{pmatrix} \cos \theta(x) & \sin \theta(x) \\ \sin \theta(x) & -\cos \theta(x) \end{pmatrix} \right] & \psi_i(x) = E \psi_i(x), \\
& i = \uparrow, \downarrow,
\end{align*}
\]

where \( \Delta \) is the energy splitting between carriers of opposite spin orientation in the ferromagnetic material.

This Schrödinger equation cannot be analytically solved for the \( \theta(x) \) shown in figure 1(b). We approximate the magnetization inside the domain walls as a piecewise linear function, such that \( \theta_i(x) = \theta_{i-1}(x) + \phi_i(x - x_i)/d_i \), where \( \phi_i \) is the total magnetization rotation in segment \( i \), \( x_i \) is the leftmost position of the segment, and \( d_i \) is the width of the segment. We thus split each domain wall into 17 segments, each a uniform spin spiral, with the largest number of segments concentrated on the outer tails of the domain walls, where the change in magnetization with position is less linear. We find that the specific number (and therefore size) of the segments used to split up the domain wall does not significantly affect the calculations; we find the same results for 50 equal segments. For each linear domain wall segment, we solve equation (2) by transforming to a rotating basis [51, 52]. The rotation matrix

\[ R_i = e^{-i \frac{\phi_i}{2} \sigma_z} = \begin{pmatrix} \cos \frac{\phi_i}{2} & -\sin \frac{\phi_i}{2} \\ \sin \frac{\phi_i}{2} & \cos \frac{\phi_i}{2} \end{pmatrix} \]

defines \( \psi_i = R_i \psi_i \) and removes the \( \theta \) dependence from the off-diagonal potential matrix,

\[ R_i^{-1} \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix} R_i = \sigma_z, \]

and yields a modified Schrödinger equation

\[ \left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{i \hbar^2 \phi_i}{2m^* d_i} \frac{\partial}{\partial x} - \frac{\Delta}{2 d_i^2} \sigma_z + \frac{\hbar^2 \phi_i^2}{8m^* d_i^2} \right] \varphi_i = \frac{E}{d_i} \varphi_i. \]

For coherent transport through a uniform spin spiral, analytic solutions for spin-dependent wave function coefficients are possible in each piecewise-linear segment of the domain wall. We use transfer matrices to connect the wavefunctions in each linear segment, and then solve for the total reflection and transmission coefficients, with and without spin flip, as well as the wavefunctions for the central domain.

3. Spin transport through the two domain wall system

After obtaining the full wavefunctions for the entire system, we define a spin current density [53]

\[ Q_i = \frac{\hbar}{2m^*} \left[ \psi_i^\dagger S (\partial_x \psi_i) - (\partial_x \psi_i^\dagger) S \psi_i \right]. \]

The tensor \( Q \) has a flow direction in real space as well as a direction in spin space. As our transport model is one-dimensional, the real-space flow direction lies solely along the \( \hat{x} \) direction, and we write \( Q \) as a vector with components corresponding to the appropriate spin-space directions. As this spin current is not a conserved quantity, we can then define the spin torque per unit area as the amount of spin current lost to the domain wall during transport [53]

\[ \mathbf{N}_{DW} = Q_x - Q_y. \]
The total spin torque is then calculated by integrating the transmission and reflection coefficients over the carrier population. As in [41], we assume the left and right hand ferromagnetic regions to be reservoirs of spin-polarized carriers, with chemical potentials \( \mu_L = eV \) and \( \mu_R = 0 \).

Thus the spin torque caused by injection of carriers from the left side \( N_L \) is a function of the majority population on the left side of the domain wall, and similarly on the right:

\[
N_L = N_f L \tag{8}
\]

\[
N_R = -N_f R \tag{9}
\]

The total spin torque is then the sum of the the torque from the left side, and the torque from the right side, integrated over the carrier population:

\[
N_{tot} = \int d^3k (N_L + N_R) = \int d^3k [N(f_{Lmaj} - f_{Rmaj})], \tag{10}
\]

where the applicable distribution functions are

\[
f_{Lmaj,Rmaj} = \frac{1}{1 + e^{(\mu - \mu_L)/k_B T}}. \tag{11}
\]

Calculations shown here are for a pair of 1.8 nm thick domain walls, based on earlier calculations [41] showing domain walls this size experience non-negligible spin torque. Although small, domain walls in the 1–10 nm range exist in many systems [47, 54, 55]. We consider a material with a spin splitting of 100 meV (same as in [47]) and an effective carrier mass of \( 0.45 m_e \) [56, 57], where \( m_e \) is the mass of the bare electron. The calculations are performed with a temperature of 110 K, and the carrier density is chosen to be \( \sim 10^{29} \text{ cm}^{-3} \). This gives a Fermi wavelength of 9.4 nm and a Fermi Energy of 37.6 meV. These parameters are representative of a magnetic semiconductor (without spin–orbit interaction), and correspond to those used in [41] for ease of comparison, but the general qualitative features found here do not depend on the detailed quantitative parameters chosen. Calculations performed for a range effective masses \( (0.1 m_e – 1.0 m_e) \) and spin splittings \( (10 \text{ meV} – 600 \text{ meV}) \) showed that the results scale with the width of the domain walls. The effect of temperature in these calculations is to increase the distribution of majority-spin carriers that experience large spin precessions upon moving through the domain wall, because they have larger momentum, and hence increase the amount of spin torque. The spin torque is small at low temperature, but increases substantially as the temperature is elevated to 100 K. Eventually when the temperature becomes comparable to the spin splitting, the torque produced will be reduced because torque generated by one spin direction of carrier will be cancelled by the torque generated by the opposite-spin carrier with the same momentum.

**4. Spin torque and interaction of the two domain walls**

Figure 2 shows calculated probabilities for transmission and reflection of the carriers, with and without spin flip, for these two domain walls, for a set of four domain wall separations. At a distance of 10 nm (a), we see a transmission spectrum with a number of resonance peaks in the energy region below the spin-splitting \( \Delta \), and largely transmission without any spin flip above \( \Delta \). This behavior is that of a spin-dependent double barrier resonant tunneling structure. At a distance of 5 nm (b), the resonant behavior remains, with fewer peaks, and some spin-flipped transmission at energies above \( \Delta \). At a distance of 1 nm (c), the peaks are even fewer, and there is additional transmission away from the resonance peaks for energies
below $\Delta$ as the domain walls are getting close enough to both be interacting with the spin current at the same time. A number of carriers with energies above $\Delta$ are reflected with flipped spins at this separation. When the two walls are brought into contact with each other (d), the transmission spectrum has many of the features of a single $2\pi$ wall (inset, similar to [41]).

The calculated energy-resolved components (torkance) of the total spin torque acting on both walls from equation (7) are shown in figure 3. We identify the spin torque as adiabatic (proportional to $\nabla M(r)$, and thus parallel to $\hat{x}$ or $\hat{z}$) or non-adiabatic (proportional to $M(r) \times \nabla M(r)$, parallel to $\hat{y}$).

We see again the resonant behavior for large separations of the domain wall. However, the spikes in the individual components are offset from each other. The resonant spikes in the $\hat{x}$ component of the torque correspond to energies where the magnitude of the $\hat{y}$ component of the torque is at a minimum. This is due to the different causes of the two kinds of torque. The adiabatic torque is the result of carriers being transmitted after performing the necessary spin flips to align with each downstream domain. The non-adiabatic torque is caused by mistracking, as carriers that do not make the full spin rotation are reflected. This resonant behavior

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Figure 2. Probabilities for transmission and reflection with and without spin flip, for a pair of 1.8 nm thick Néel $\pi$ walls with separation between them of (a) 10 nm, (b) 5 nm, (c) 1 nm, and (d) 0 nm. Inset: the same transmission/reflection spectrum for a 3.6 nm thick $2\pi$ wall.

Figure 3. Spin torque as a function of energy, for a pair of 1.8 nm thick Néel $\pi$ walls with separation between them of (a) 10 nm, (b) 5 nm, (c) 1 nm, and (d) 0 nm. Inset: the same calculation for a 3.6 nm thick $2\pi$ wall.
decreases as the separation between the domain walls decreases, reducing to a spin torque profile similar to that of a $2\pi$ wall when the domain walls are brought into contact. The behavior of the $\hat{z}$ component of the torque remains consistent, and is zero until the energy rises above the spin splitting $\Delta$.

Figure 4 shows the total spin torque for the double wall system as a whole, as a function of the separation of the two walls, and compares it to the spin torque in individual $\pi$ and $2\pi$ walls. We find that the the spin torque in the direction of the motion ($\hat{x}$) increases as the two walls move a small distance away from each other, and then falls to a saturation level.

Figure 5. Spin torque as a function of domain wall separation, for each of two 1.8 nm thick Néel $\pi$ walls. Solid lines are torque for the upstream wall, dashed lines for the downstream wall, as shown in figure 1(a).
that is higher than the spin torque in a $2\pi$ wall double the size of one $\pi$ wall and higher than twice the spin torque in an individual $\pi$ wall. The spin torque in the $\hat{y}$ direction has its largest magnitude at a smaller separation, and saturates to a level very close to that of a $2\pi$ wall or two times that of a single $\pi$ wall. There is no significant spin torque in the $\hat{z}$ direction for the system as a whole.

Figure 5 shows the breakdown of the spin torque for each individual domain wall as a function of the separation of the two walls. The velocity of each one of these domain walls is determined by the spin torque in the $\hat{z}$ direction, and can be estimated as

$$v = \frac{g\mu_b N_z}{M_s}$$

(12)

where $N_z$ is the $z$-component of the spin torque in $h$ flips per second per cm$^2$ and $M_s$ is the saturation magnetization of the material. As this is directly proportional to the spin torque, this picture of the torque represents the motion of each domain wall.

We also check to see if the relationship between the spatially-dependent torque and the magnetization follows the velocity, or if an internal rigidity of the domain wall is required to justify our calculations [58]. The result is shown in figure 6. We find that $N(x)$ and the derivative of $M(x)$ are approximately proportional inside the domain walls, especially for the component along the $\hat{z}$ direction, suggesting that the motion of the domain walls induces minimal internal domain wall compression or tension (if damping in the domain walls can be neglected) [18, 59, 60].

While the total amount of spin torque in the $\hat{z}$ direction, shown in figure 4(c), is zero, the torque for each individual wall, shown in figure 5(c), is not. The spin torques for the upstream and downstream walls are symmetric, which leads to them having the same velocity. When the domain walls are far apart, they move independently at a relatively high constant speed. That speed decreases considerably when the domain walls are brought close together. This suggest a lower limit for the spacing of domain walls on a ‘racetrack’ [33]. Placing the domain walls closer together to decrease the bit size significantly decreases the speed of the domain walls.

The spin torque in $\hat{y}$ direction (coming again from mis-tracking carriers), shown in figure 5(b) attempts to pull the domain wall out of its Néel configuration. This effect is at its largest when the domain walls are very close to each other, with the upstream wall experiencing a large negative torque,
and the downstream wall experiencing very little torque. As the walls separate, the magnitude of the torque decreases in the upstream wall and increases in the downstream wall, until they saturate to a constant value. Because of the magnetic anisotropy of the Néel wall system, this torque does not contribute to the velocity of the domain walls.

The spin torque in the \( \hat{x} \) direction would tend to cause flexing in the domain walls themselves. Figure 5(a) shows that each wall experiences a unique non-zero amount of this torque. As we are considering the motion of rigid walls here, this torque also does not contribute to the velocity of the domain walls.

5. Concluding remarks

We have shown that two domain walls separated by some distance exhibit a nonlinear relationship in their interaction when the walls are very close together. This indicates an ideal spacing where we could manipulate two or more domain walls to move in a stable configuration along a track with the application of a single current.

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