Electric-field manipulation of the Landé $g$ tensor of a hole in an In$_{0.5}$Ga$_{0.5}$As/GaAs self-assembled quantum dot

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The effect of an electric field on spin precession of a hole in an In$_{0.5}$Ga$_{0.5}$As/GaAs self-assembled quantum dot is calculated using multiband real-space envelope-function theory. The dependence of the Landé $g$ tensor on electric fields should permit high-frequency $g$-tensor modulation resonance, as well as direct, nonresonant electric-field control of the hole spin. Subharmonic resonances have also been found in $g$-tensor modulation resonance of the hole, due to the strong quadratic dependence of components of the hole $g$ tensor on the electric field.

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I. INTRODUCTION

Since the initial proposals of spin-based quantum-information processing in solids, methods of controlling individual spins in a scalable fashion have drawn considerable attention. An especially attractive method for independently controlling spins spaced much closer than an optical spot size is to locally modify some of the spin’s properties with an electric field. This approach’s advantages include well-developed techniques for generating large arrays of independently controllable electric gates. Initial proposals focused on modifying the resonant frequency of a spin in a magnetic field using spatially dependent factors or hyperfine fields. This approach requires a spatially extended always-on microwave field with which local spins are brought in and out of resonance. An alternate approach replaces electric-field control of a resonance frequency with electric-field control of the magnetic or pseudomagnetic field experienced by the spin, which can produce spin resonance when the electric field is oscillated at the resonance frequency. Electric-field control of the magnetic field has been demonstrated by moving an electron back and forth (using electric-field control of the location of quantum confinement of a quantum dot) in an inhomogeneous magnetic field. Electric-field control of a pseudomagnetic field has focused on modifying the orbital moment of an electron, which can affect the spin through the spin-orbit interaction. One such mechanism drives the spin through the $g$ tensor ($g$-tensor modulation resonance) and has been demonstrated in quantum wells and also proposed for electron spin manipulation in quantum dots, for donor-bound electrons, and for holes in quantum dot molecules. Another mechanism works directly through the spin-orbit entanglement of the electronic wave function and has been demonstrated for electron spins in dots and described for hole spins in dots.

Advantages of using hole spins instead of electron spins for $g$-tensor modulation resonance include the larger spin-orbit interaction and orbital angular momenta in the valence band, the stronger dependence (and asymmetry) of hole $g$ tensors on structural shape, and possibly reduced hyperfine interactions. The larger spin-orbit interaction for holes and larger orbital angular momenta lead to larger hole $g$-tensor components. If the fractional change in orbital angular momentum with applied electric field is comparable for electrons and holes, then the variation in $g$-tensor components should be much larger for holes, leading to much more rapid spin manipulation.

As shown in Ref. 8, full $4\pi$ manipulation of an electron spin may be obtained using a series of vertical gate voltages if one component of the $g$ tensor changes sign as a function of the applied electric field, which can be achieved with a highly anisotropic dot shape. The orientations of the fields are shown schematically in Fig. 1. The electric field direction is fixed by the gate, and the magnetic field direction is chosen so the sign change in one component of the $g$ tensor leads to a $90^\circ$ rotation of the spin precession axis. Zero in-plane $g$-tensor components were predicted for holes in self-assembled InAs/GaAs quantum dots with a circular footprint, although once the footprint becomes elliptical these $g$-tensor components grow rapidly. Other calculations have shown nonzero in-plane $g$-tensor components in Ge/Si (Ref. 17) and III-V nanowhisker quantum dots as well as in a two-dot InAs/GaAs quantum dot molecule. Experimental measurements have seen small, but nonzero in-plane hole $g$ factors in InP/InGaP, CdSe/ZnSe, and InGaAs/GaAs quantum dots. As an extreme example, in a Ge/Si nanowire quantum dot, electrical control of the $g$-tensor component parallel to the wire has been experimentally realized by varying the voltage on the electrostatic gates which define the dot. Another extreme asymmetric structure is the vertically -coupled quantum-dot molecule, in which efficient hole $g$-tensor modulation by electric fields has been predicted, including the possibility of spin echo.

Hole spins have very short spin relaxation times in bulk, due to angular-momentum mixing and state degeneracy typically found in the valence band, but this time increases in quantum dots up to the same order of magnitude as electron spin relaxation times. Because the hole Bloch functions have $p$-like character, the contact term of the hyperfine coupling to the nuclear spin is zero, leaving higher-order, weaker, anisotropic terms. Recent measurements have found hole $T_2$ times of $\sim 100$–1000 ns in the presence of an in-plane magnetic field. Theoretical estimates of the hyperfine interaction in GaAs dots suggest $T_2$ times of order 40 ns for growth-direction magnetic fields, whereas experimental measurements on...
InGaAs dots suggest a hyperfine interaction strength a factor of 2 smaller than estimated for GaAs dots,\textsuperscript{30} yielding a similar \(\sim\)100-ns time scale. Lengthening of the \(T_2\) time by nuclear spin state narrowing\textsuperscript{31} is also possible. Hole \(T_1\) times are longer than these \(T_2\) times and exceed 1 \(\mu\)s for magnetic fields up to 10 T.\textsuperscript{32,33} In order to make a quantum computer, at least \(10^{10}\) operations must be performed during the decoherence time.\textsuperscript{34} Thus these \(T_2\) values require a maximum spin manipulation time of the order of 10–100 ps.

Here we predict that spin manipulation using a static magnetic field and a time-varying electric field is possible for a hole in a single In\(_{0.5}\)Ga\(_{0.5}\)As/GaAs quantum dot using a single vertical electrical gate, with an electric-field oscillation frequency either resonant or nonresonant with the Larmor frequency, and that the spin manipulation times are more rapid than those for electron spins in the same quantum dots. We also find that the nonlinearity of the \(g\)-tensor components with applied electric field is highly anisotropic, with a much stronger quadratic electric-field dependence of the \(g\)-tensor component parallel to the electric-field direction than perpendicular to it. Such quadratic electric-field dependencies have previously been studied for systems of high symmetry, like donor states,\textsuperscript{9} for which the linear electric-field dependence vanishes. Here we find that these nonlinear \(g\)-tensor components generate highly anisotropic \textit{subharmonic} resonances which effectively manipulate the hole spin. For magnetic fields of \(\sim 5–10\) T, the spin manipulation times are of the order of 20–30 ps for electric fields \(\sim 150–200\) kV/cm.

II. HOLE \(g\)-TENSOR COMPONENTS IN A SINGLE QUANTUM DOT

A. Theoretical method

We have computed \(g\) tensors for a hole confined in a lens-shaped In\(_{0.5}\)Ga\(_{0.5}\)As/GaAs self-assembled quantum dot as a function of an electric field applied along the growth direction and ranging from \(-150\) to 150 kV/cm. The dot has a height of 6.2 nm, a 21.6-nm base along the minor axis ([110] direction) and 32.8-nm base along the major axis ([1\(\bar{1}\)0] direction). The states were calculated using an eight-band \(k \cdot p\) strain-dependent Hamiltonian\textsuperscript{35} in the envelope approximation using finite differences on a real-space grid.\textsuperscript{36} The strain is considered in all three dimensions. This method has been used previously to calculate \(g\) tensors of electrons in both quantum dots\textsuperscript{8,14} and bound to donors.\textsuperscript{9}

The large electric fields considered here, when applied to the dot/barrier system we consider in this paper, would ionize an electron;\textsuperscript{8} however, a hole will remain confined due to its large effective mass. Whereas bulk holes have \(J = 3/2\), in a self-assembled dot the geometric asymmetry and strain break the fourfold degeneracy, resulting in doubly degenerate levels that are mixtures of heavy and light holes. Although the lowest-energy hole state is a doublet (mostly heavy hole, with small light-hole components), none of the eight envelope functions is identically zero and therefore the doublet energies will be split by a magnetic field pointing in any direction.

The hole \(g\)-tensor component for a magnetic field \(B\) applied in a direction \(\hat{d}\) was found by calculating the splitting \(\Delta E\) to obtain \(|g_\alpha| = \Delta E/3\mu_B B\), where \(\mu_B\) is the Bohr magneton. We follow the convention for holes of including a factor of 3 in the denominator (reflecting \(m_f = \pm 3/2\)) even though the states form a two-level system and are not pure heavy hole. The sign of \(g_\alpha\) was determined from the spin orientation of the ground-state wave function, with \(g_\alpha > 0\) for the spin pointing antiparallel to \(B\). Since the lens-shaped dots were elongated along the [110] direction the principal axes were [110], [1\(\bar{1}\)0], and [001]. Spin splittings were computed for \(B\) along each of these symmetry directions to obtain each \(g_\alpha\).

The ground-state hole wave functions are shown in Fig. 2. As expected, the ground state is primarily heavy hole and is well confined to the quantum dot. The wave function is only slightly affected by the applied electric field due to the strong dot confinement.

B. \(g\)-tensor dependence on electric field

Earlier investigations of electron spin manipulation using an electrically controlled \(g\) tensor identified the importance of having a \(g\)-tensor component that changes sign with electric field.\textsuperscript{8} If the sign of one \(g_\alpha\) can be changed with an applied electric field, then it is possible to precess the spin to point in an arbitrary direction by only changing the electric field (in the presence of a suitably oriented static magnetic field). The dot with the geometry shown in Figs. 1 and 2 has this desired sign change. The \(g\) tensor of this dot as a function of electric field is plotted in Fig. 3.

The most interesting component is \(g_{[001]}\) which is concave upward with a minimum value \(g_{[001]} \approx -0.1\) at \(E \approx 0\) and \(g_{[001]} \approx 0\) at \(E \approx \pm 100\) kV/cm. Therefore, there are two possible electric-field values around which nonresonant spin manipulation may be done over the entire Bloch sphere as described in Ref. 8. This double sign change limits the range over which such spin manipulation may be performed, as the limited range of \(|g_{[001]}|\) constrains the effective spin precession frequency. However, the field ranges involved correspond roughly to the breakdown fields in GaAs, so larger electric fields are unlikely to be practical.

In Fig. 3 we see a significant nonlinearity in \(g_{[001]}\), whereas the nonlinearities in \(g_{[110]}\) and \(g_{[1\bar{1}0]}\) are much smaller. These nonlinearities were parametrized by fitting a second-order polynomial in \(E\) to the \(g\)-tensor components, resulting in the coefficients given in Table I.
Although the detailed trends associated with the electric-field dependence of the $g$-tensor components are challenging to trace to specific characteristics of the dot, some qualitative statements can be made. Positive and negative electric fields push the hole closer to the apex and base of the dot, respectively. Any increase in the magnitude of $E$ will increase confinement, driving $g$ toward 2. This mechanism explains $g_{[001]}(E)$ (see Fig. 3) because the states in a [001]-oriented magnetic field may be formed primarily out of the heavy-hole ground-state doublet. However, the in-plane hole $g$-tensor components depend most strongly on the possible mixing due to an applied magnetic field between the lowest-energy heavy-hole–light-hole doublet in the dot and the lowest-energy light-hole doublet, which permits the formation of an in-plane hole spin orientation. The inhomogeneous strain in the dot causes a spatial variation in the heavy-hole–light-hole splitting. As the electric field moves the hole within the dot, the effective heavy-hole–light-hole splitting changes, providing an additional contribution to $g_{[001]}$. The complicated interaction of these two effects gives rise to the monotonic behavior of $g_{[110]}$ and $g_{[110]}$ with $E$ seen in Fig. 3.

We find the concave-up shape of $g_{[001]}$ in Fig. 3 is common to a wide range of dots (and is shared by the [001] electron $g$-tensor component as well\(^5\)); the necessary criterion for use in nonresonant $g$-tensor control is that it changes sign for an experimentally feasible electric field. The shape and size of an elliptical lens can be described by three parameters [the lengths of the three principal axes, or equivalently the height, elongation (ratio of the two principal axes of the footprint), and optical transition energy]. For dots which are not highly elongated (ratio of the two principal axes of the footprint), the footprint length along the [110] direction of 21.6 nm, and a footprint length along the [10] direction of 32.8 nm. Of particular note is the sign change of the $g$-tensor component along the [001] direction.

### Table I. Coefficients of a fit of the $g$ tensor in Fig. 3 to $g_{\alpha} = \hat{a}_\alpha E^2 + \hat{b}_\alpha E + \hat{c}_\alpha$, where $E$ is the electric field component along [001].

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>[001]</th>
<th>[110]</th>
<th>[10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>-0.115</td>
<td>0.143</td>
<td>0.208</td>
</tr>
<tr>
<td>$b$ (cm/kV)</td>
<td>$1.15 \times 10^{-4}$</td>
<td>$1.88 \times 10^{-4}$</td>
<td>$2.52 \times 10^{-4}$</td>
</tr>
<tr>
<td>$a$ (cm$^2$/kV)</td>
<td>$1.23 \times 10^{-5}$</td>
<td>$5.20 \times 10^{-7}$</td>
<td>$5.08 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
asymmetric, if two of the parameters in either set of three above are fixed, then the other one can be set by the requirement that \( g_{[001]} \) change sign for an electric field between \(-100\) and \(+100\) kV/cm. The component \( g_{[001]} \) is not even with \( E \); this is due to the reflection asymmetry of the dot geometry, enhanced by the dot strain. A similar asymmetry in \( g_{[001]} \) was found for electrons in Ref. 8.

Quantum dots may have compositional gradients in the growth direction; these affect our results by (1) changing the effective height of the dot and (2) changing the effective electric field acting on a hole in the dot when there is no externally applied electric field. For the first case the effects can be understood from calculations of the height dependence of hole \( g \) tensors,\(^{34}\) and only involve a small quantitative shift of the \( E = 0 \) \( g \) tensor. The second case simply results in an offset for the effective \( E \) depending on the details of the indium concentration gradient.

III. SPIN MANIPULATION USING \( g \)-TENSOR MODULATION WITH AN ELECTRIC FIELD

A. Nonresonant hole spin manipulation with an electric field

We first consider nonresonant spin precession using the technique developed in Ref. 8. By applying two different electric fields \((E_1\) and \(E_2\)) along the growth direction, precession around two orthogonal axes may be obtained. This approach requires the \( g \)-tensor component along one symmetry direction (here \([001]\)) to change sign as a function of \( E \). For a given \( E_1 \) and \( E_2 \) such that \( g_{[001]}(E_1)g_{[001]}(E_2) < 0 \), the magnetic field direction required to obtain orthogonal spin precession axes \( \Omega = \mu_B \hat{g} \cdot \mathbf{B}/\hbar \) is determined by the condition

\[
(\hat{g}(E_1) \cdot \mathbf{B}) \cdot (\hat{g}(E_2) \cdot \mathbf{B}) = 0.
\]

The optimal solution is determined by maximizing \( |\Omega(E_1)| \) subject to \( |\Omega(E_1)| = |\Omega(E_2)| \) and \(|E_1|,|E_2| < 150 \) kV/cm (to avoid breakdown). For the dot geometry corresponding to Fig. 3 we obtain \( E_1 = -150 \) kV/cm and \( E_2 = 3.1 \) kV/cm. The optimal magnetic field angle \((0.24\pi)\) is nearly \(\pi/4\), measured from the \([001]\) axis toward the \([1\overline{1}0]\) axis. For a magnetic field of \(5 \) T the time for the spin to precess by \(\pi\) is 18 ps.

B. Resonant hole spin manipulation with an electric field

For a \( g \) tensor that depends linearly on the electric field, resonances occur when the oscillation frequency of the electric field match the Larmor frequency of the spin in the static magnetic field. However, when the \( g \) tensor depends nonlinearly on the electric field, resonances may occur at subharmonics of the Larmor frequency.\(^{9}\) The strongly nonlinear behavior of the \( g \) tensor for holes, evident in Fig. 3 and parametrized in Table I, produces such subharmonic resonances. For an applied field \( E(t) = E_{dc} + E_{ac} \sin(\omega t) \), the response amplitudes are

\[
\Omega_{\alpha}(t) = (\mu_B/\hbar)B_\alpha(a_\alpha E^2(t) + b_\alpha E(t) + c_\alpha),
\]

\[
= \Omega_{\alpha,0} + \Omega_{\alpha,1} \sin(\omega t) - \Omega_{\alpha,2} \cos(2\omega t),
\]

where

\[
\Omega_{\alpha,0} = (\mu_B/\hbar)B_\alpha \left[ E_{dc}^2 + E_{ac}^2/2 \right] a_\alpha + E_{dc} b_\alpha + c_\alpha,
\]

\[
\Omega_{\alpha,1} = (\mu_B/\hbar)B_\alpha E_{ac} \left[ 2E_{dc}a_\alpha + b_\alpha \right],
\]

\[
\Omega_{\alpha,2} = (\mu_B/\hbar)B_\alpha E_{ac}^2 a_\alpha/2.
\]

\( \Omega_1 \) is the response at the fundamental \( \omega = \omega_0 \) and \( \Omega_2 \) is the response at the subharmonic \( \omega = \omega_0/2 \), where \( \omega_0 \) is the Larmor precession frequency. Higher-order polynomial dependencies (e.g., cubic or quartic) of the \( g \) tensor on the electric field will result in resonances at additional subharmonics of the Larmor frequency. Higher-order effects, including the counterrotating components of the oscillating transverse component of the spin precession vector, will bring additional shifts of the lower-order resonances and bring in resonances at other multiples of the Larmor frequency.\(^{37}\)

The precession rates for the subharmonic and fundamental resonances were calculated to first order for the full range of magnetic field angles, and for electric field amplitudes less than the breakdown field \(\sim 200 \) kV/cm. The Rabi frequency associated with the fundamental resonance was found for an electric field oscillating about \( E_{dc} = 0 \) as a function of applied magnetic field direction, \( \phi \), in the \([001]-[1\overline{1}0]\) plane and as a function of electric field amplitude \( E_{ac} \). For any given value of \( E \), an optimal magnetic field direction was found, corresponding to the largest Rabi frequency. As a function of \( E \), the optimal magnetic field direction increases monotonically. At \( 200 \) kV/cm the largest Rabi frequency for a magnetic field of \(10 \) T is \( 18 \) GHz at an optimal magnetic field angle of \(1.2\) rad from the \([001]\) axis. The time required for the spin to precess by \(\pi\) in this configuration is \(\sim 28\) ps.

The Rabi frequency for the subharmonic resonance was also found for an electric field oscillating about \( E_{dc} = 0 \) as a function of applied magnetic field direction \( \phi \) and electric field oscillation amplitude \( E_{ac} \) and is plotted in Fig. 4 for \( B = 1 \) T. As with the fundamental resonance, a general trend of faster spin manipulation times at higher \( E \) was found through \( E = 200 \) kV/cm, peaking at an angle of \(1.2\) rad from the \([001]\) axis. The \( g \) tensor for this dot was found to be linear in the magnetic field up to at least \( B = 10 \) T; at \( B = 10 \) T the peak Rabi frequency was \(39\) GHz, corresponding to a minimal time for the spin to precess by \(\pi\) of \(13\) ps.

![FIG. 4. (Color online) Contour plot of Rabi frequency (in GHz) at quadratic resonance as a function of ac electric-field amplitude \( E_{ac} \) and magnetic-field angle, for \( B = 1 \) T. \( E_{dc} = 0 \).](image-url)
IV. CONCLUDING REMARKS

We have examined theoretically the hole $g$ tensors in In$_{0.5}$Ga$_{0.5}$As/GaAs quantum dots for possible application to hole spin manipulation. A structure was proposed for which one $g$-tensor component sign changed as a function of electric field applied along the [001] growth direction. The nonlinear $g$-tensor dependence on applied electric field causes a subharmonic resonance to appear at $\omega = \omega_0/2$, with higher-order dependencies generating further subharmonics. The $g$-tensor for this structure was used to calculate resonant and nonresonant spin manipulation frequencies. For a magnetic field of 10 T, the resonant spin manipulation method had an optimal spin manipulation time (precession by $\pi$) of 28 ps at the fundamental resonance or 13 ps at the subharmonic resonance. The nonresonant spin manipulation time for a 5-T field was 18 ps. Using the experimental values of $T_2$ from Ref. 29 or estimates inferred from Refs. 27 and 30, approximately $10^4$ operations would be possible during a $T_2$ time.

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