Electric-field dependent spin diffusion and spin injection into semiconductors

Z. G. Yu and M. E. Flatté
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242
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We derive a drift-diffusion equation for spin polarization in semiconductors by consistently taking into account electric-field effects and nondegenerate electron statistics. We identify a high electric-field diffusive regime which has no analog in metals. In this regime there are two distinct spin-diffusion lengths. Furthermore, spin injection from a ferromagnetic metal into a semiconductor is enhanced by several orders of magnitude. This enhancement also occurs for high-field spin injection through a spin-selective interfacial barrier.

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Semiconductor devices based on the control and manipulation of electron spin (semiconductor spintronics) have recently attracted considerable attention.1 Spin transport and injection properties of semiconductors and heterostructures strongly constrain the design of new spintronic devices. In theoretical studies of spin transport and injection in semiconductors2–4 the spin polarization is usually assumed to obey the same diffusion equation as in metals,5

\[ \nabla^2(\mu_s - \mu_d) - (\mu_s - \mu_d)/L^2 = 0, \tag{1} \]

where \( \mu_s \) is the chemical potential of up-spin (down-spin) electrons. In this diffusion equation the electric field does not play any role, and spin polarization decays away on a length scale of \( L \) from an injection point. This is reasonable for degenerate systems because the electric field \( E \) is essentially screened. For semiconductor spintronic devices, however, the semiconductor often is lightly doped and nondegenerate, and experiments have shown the electric field can change spin diffusion dramatically.6,7 Equation (1) corresponds to neglecting drift in the more general drift-diffusion equation for the spin polarization,

\[ \nabla^2(n_u - n_d) + \frac{eE}{k_BT} \nabla(n_u - n_d) - \frac{(n_u - n_d) / [L^{(s)}]^2}{10} = 0, \tag{2} \]

where \( n_u - n_d \) is the difference between up-spin and down-spin electron densities and \( L^{(s)} \) is the intrinsic spin-diffusion length. For degenerate systems \( k_BT \) should be (approximately) substituted by \( E_F \) (the Fermi energy) in Eq. (2), and Eq. (1) is thus justified for metals.

If Eq. (1) were to hold for semiconductors, spin injection from a ferromagnetic metal to a semiconductor without a spin-selective interfacial barrier would be virtually impossible due to the “conductivity mismatch,” or more precisely, a mismatch between effective resistances in the metal \( L^{(f)}/\sigma_f \) and in the semiconductor \( L^{(s)}/\sigma_s \).2–4 Here \( L^{(f)} \) and \( L^{(s)} \) are the spin-diffusion lengths for the ferromagnetic metal and the semiconductor, and \( \sigma_f \) and \( \sigma_s \) are conductivities for the two materials. Even for spin injection from ferromagnetic semiconductors, \( L^{(f)}/\sigma_f < L^{(s)}/\sigma_s \), and the spin polarization \( p_s < 99 \% \), so the large spin injection percentages achieved from ZnMnSe (Refs. 8 and 9) and GaMnAs (Ref. 10) are difficult to understand via Eq. (1).

Here we clarify the central role of the electric field on spin transport in semiconductors. We obtain the drift-diffusion equation for the spin polarization in a semiconduc-

tor, Eq. (2), which consistently takes into account electric-field effects and nondegenerate electron statistics. We identify a high-field diffusive regime which has no analog in metals. This regime occurs for fields as small as 1 V/cm at low temperatures. Two distinct spin-diffusion lengths now characterize spin motion, i.e., upstream \( L_u \) and downstream \( L_d \) spin diffusion lengths, which can differ in orders of magnitude with realistic fields: \( E \approx 10 \) V/cm at \( T = 3 \) K and \( E \approx 10^3 \) V/cm at \( T = 300 \) K. These two length scales play distinctive but both favorable roles in spin injection from a ferromagnetic metal to a semiconductor. We further find that the effective semiconductor resistance determining the injection efficiency is \( L_d / \sigma_s \) rather than \( L^{(s)}/\sigma_s \), which may be comparable to \( L^{(f)}/\sigma_f \) given that \( L_u \) can be shorter than \( L^{(s)} \) by several orders of magnitude in the high-field regime. Moreover, the decay length scale for the spin polarization injected into the semiconductor is \( L_d \), which would be much longer than \( L^{(s)} \) in this regime.11 Our results suggest a practical approach to increase spin injection into semiconductors by orders of magnitude: by increasing the electric field, or equivalently, increasing the total injection current in semiconductors. Our results are consistent with the significant current dependence observed for spin injection from Fe to GaAs.12 We further note that strong fields also substantially enhance spin injection in structures with an interfacial barrier.

The semiconductor we consider here is lightly or moderately \( n \)-doped \((p\)-doped semiconductors can be analyzed similarly), which is often encountered in spintronic devices. We assume that there is no space charge and the material is homogeneous. The current for up spin and down spin can be written as \( j_{1(1)} = \sigma_{1(1)} E + eD \nabla n_{1(1)} \), which consists of the drift current and the diffusion one. Here \( D \) is the electron diffusion constant, \( \sigma_{1(1)} \) the up-spin (down-spin) conductivity, and \( n_{1(1)} \) the up-spin (down-spin) electron density. The spin-dependent conductivity is proportional to the electron density for individual spins, \( \sigma_{1(1)} = \eta_{1(1)} e \nu_e \). The rate at which spin-up (spin-down) electrons scatter to spin-down (spin-up) electrons is denoted by \( 1/\tau_{1(1)} \). In a steady state, the equations of continuity for individual spins read

\[ \nabla \cdot j_1 = \nabla \sigma_1 \cdot E + \sigma_1 \nabla \cdot E + eD \nabla^2 n_1 = \left( \frac{n_1}{\tau_{1(1)}} - \frac{n_1}{\tau_{1(1)}} \right) e, \tag{3} \]

\[ \nabla \cdot j_1 = \nabla \sigma_1 \cdot E + \sigma_1 \nabla \cdot E + eD \nabla^2 n_1 = \left( \frac{n_1}{\tau_{1(1)}} - \frac{n_1}{\tau_{1(1)}} \right) e, \tag{4} \]
where $\nabla \cdot \mathbf{E} = -e \Delta n/e$ and $\Delta n$ is a local variation of electron density. In nondegenerate semiconductors, $\tau^{-1} = \tau_1^{-1} = \tau^{-1/2}$. Generally speaking, a strong electric field ($\gg 10^5 \text{ V/cm}$) may modify $\tau$, $\nu_e$, and the band structure of semiconductors. However, as we will show later, the electric field even in the “high-field” regime is still relatively weak ($\ll 10^5 \text{ V/cm}$) and we assume that both $\nu_e$ and $\tau$ are independent of the field.

For a homogeneous semiconductor without space charge, $\Delta n$ should be roughly balanced by a change in hole density $\Delta p$. In doped semiconductors, however, spin polarization can be created without changing electron or hole densities, and therefore,

$$\Delta n_1 + \Delta n_1 = 0.$$  \hspace{1cm} (5)

Here $\Delta n_{1(1)} = n_{1(1)} - n_0/2$, and $n_0$ is the total electron density in equilibrium. Care is required, however, to avoid setting $\nabla \cdot \mathbf{E} = 0$ directly in Eqs. (3) and (4). Instead we multiply Eq. (3) by $\sigma_j$ and Eq. (4) by $\sigma_1$, and subtract one from the other, eliminating the terms containing $\nabla \cdot \mathbf{E}$. Only then do we set $\Delta n_1 + \Delta n_1 = 0$. By using the Einstein relation, $D = k_B T \nu_e / e$, where $k_B$ is the Boltzmann constant and $T$ is temperature, we obtain the differential Eq. (2) for $n_1 - n_1$, the measure of the spin polarization in semiconductors, with $L^{(s)} = \sqrt{D \tau}$.

Equation (2), together with the local charge neutrality constraint Eq. (5), dramatically alter the spin transport behavior in semiconductors from that expected from Eq. (1). To understand the physical consequence of the electric field on the spin transport, we suppose that a continuous spin imbalance is injected at $x = 0$, and the electric field is along the $-x$ direction. The spin polarization will gradually decay in size as the distance from the point of injection increases. In Fig. 1, we plot the spin polarization as a function of position for different fields. In the absence of the field, as shown by the solid line, the spin polarization decays symmetrically along $-x$ and $+x$ with a single length scale, $L^{(s)}$. When an electric field is applied, the decay of the spin polarization becomes spatially asymmetric. For spin diffusion opposite to the field direction (downstream for electrons), the decay length of the spin polarization is longer than $L^{(s)}$. For spin diffusion along the field direction (upstream for electrons), the decay length is shorter than $L^{(s)}$.

We define two quantities $L_d$, $L_u$,

$$L_{d(u)} = \left\{ -\left( + \frac{eE}{2k_B T} \right) + \sqrt{\frac{eE}{2k_B T} + \frac{1}{L^{(s)}}} \right\}^{-1}.$$  \hspace{1cm} (6)

The distribution of the spin polarization in Fig. 1 is then described by $n_1 - n_1 \sim \exp(-xL_d)$ for $x > 0$, and $n_1 - n_1 \sim \exp(xL_u)$ for $x < 0$. Thus $L_d [L_d > L^{(s)}]$ and $L_u [L_u < L^{(s)}]$ are the downstream and upstream spin-diffusion lengths, respectively. In the absence of the field, the downstream and upstream lengths are equal to the intrinsic diffusion length $L^{(s)}$. With increasing field the downstream diffusion length $L_d$ increases, whereas the upstream diffusion length $L_u$ decreases. A high-field regime for spin transport in semiconductors can be defined by $E > E_c$, where $E_c = k_B T = 1/L^{(s)}$. In this regime, $L_d$ and $L_u$ deviate from $L^{(s)}$ considerably and the spin-diffusion behavior is qualitatively different from that in low fields. We emphasize that since $L^{(s)}$ is large in semiconductors, this regime is not beyond realistic fields where most spintronic devices operate. For a typical spin-diffusion length, $L^{(s)} = 2 \mu$m, $E_c = 125$ V/cm at $T = 300$ K and $E_c = 1.25$ V/cm at $T = 3$ K.

The physics of the field effects on the spin diffusion becomes clearer at the strong-field limit, where $|eE|/k_B T \gg 1/L^{(s)}$. In this limit, the electrons move with velocity $|E|/\nu_e$ and so does the spin polarization. $L_d$ is simply the distance over which the carriers move within the spin lifetime $\tau$. $L_d = |E|/\nu_e \tau = |E|/(e/k_B T)D \tau = [L^{(s)}]^2 |eE|/k_B T$. For the upstream diffusion length $L_u$ at this limit, $L_u = k_B T |eE|$, which simply corresponds to a Boltzmann distribution of electrons in a retarding field.

A similar field-dependent diffusion phenomenon has been observed and studied in charge transport of minority carriers in doped semiconductors. In fact, if $n_1 - n_1$ is substituted by $\Delta p$ and $L^{(s)}$ is regarded as the intrinsic charge diffusion length, Eq. (2) becomes the diffusion equation for the disturbance of the minority carriers in $n$-doped semiconductors. It is known that the electric field leads to two distinct charge diffusion lengths in this case as well as a modification of minority-carrier injection.

As an application of our field-dependent spin transport theory, we study how the electric field affects spin injection from a ferromagnetic metal to a semiconductor. We consider a simple one-dimensional spin injection structure which, as shown in the inset of Fig. 2, comprises a semi-infinite metal ($x < 0$) and a semi-infinite nondegenerate semiconductor ($x > 0$). The depletion region between the metal and the semiconductor is assumed to be negligibly thin, for a wide depletion region is undesirable for spin transport and coherence due to the strong deviation from the local charge neutrality condition (the presence of holes dramatically shortens the electron-spin coherence time). Electrons are injected from
FIG. 2. Spin injection efficiency $\alpha_0$ as a function of electric field for structure with transparent interface. Dot-dashed, solid, short-dashed, and long-dashed lines correspond to $\sigma_f/\sigma_s = 10^2$, 10$^3$, and 10$^4$, respectively. Other parameters are $p_f = 0.5$ and $L^{(f)} = 60$ nm (appropriate for Co), and $L^{(s)} = 2$ $\mu$m (appropriate for GaAs). The inset shows the schematic injection structure. A Co/$n$-GaAs structure corresponds to $\sigma_f/\sigma_s = 10^3$. The $\sigma_f/\sigma_s = 10^2$ curve is more appropriate for injection from ferromagnetic semiconductors.

the metal to the semiconductor, and therefore the electric field is antiparallel to the $x$ axis. In the ferromagnetic metal the electrochemical potentials for individual spins satisfy the equations \[ \frac{d^2}{dx^2} \mu_s = \left( \frac{D^+_s}{\sigma_s} \right)^{-1} - \left( \frac{D^+_s}{\sigma_s} \right)^{-1} \mu_s, \]

where $D^+_s$ is the up-spin (down-spin) electron diffusion constant. In metals the conductivity and the diffusion constant are related via $\sigma_s^2 / D^+_s = 2N_s(E_F)$, where $N_s(E_F)$ is the up-spin (down-spin) density of states at the Fermi energy. It is readily seen that the above equations lead to Eq. (1) if $L^{(f)} = \left( \frac{D^+_s}{\sigma_s} \right)^{-1} + \left( \frac{D^+_s}{\sigma_s} \right)^{-1} = 1/\mu_0$.

The general solution can be written as

\[ \frac{1}{J} \mu_s = \frac{\mu_0}{\sigma_s + \sigma_s^2} \left( \frac{1}{1} + C_1 e^{-\mu_s L^{(f)}} \right) \left( \frac{1/\sigma_s^2}{1} \right), \]

where $J$ is the total electron current, which is a constant throughout the structure in a steady state. In the semiconductor, according to Eqs. (2) and (5),

\[ \Delta n_s = -\Delta n_d = C_2 \exp(-x/L_d), \]

and $J = \sigma_s E$. In order to match boundary conditions at the interface between the metal and the semiconductor, it is desirable to know the electrochemical potentials for up-spin and down-spin electrons in the semiconductor, which are related to the electron density for individual spins via

\[ \mu_s = \frac{k_B T}{n_0} \ln \left( 1 + \frac{2 \Delta n_s}{n_0} \right) + E x - C_0, \]

This relation can be readily derived based on the definition of the electrochemical potential in nondegenerate semiconductors $n_s \approx \exp[\mu_s(x) + F_0]/k_B T$, where $E = -d\psi/dx$.

The three unknown coefficients $C_i (i = 0, 1, 2)$ in Eqs. (7)–(9) will be determined by the boundary conditions at the interface. For a clean and transparent interface, i.e., no spin-flip scattering at the interface and no interface resistance, both the electrochemical potential and the current for individual spins are continuous, giving rise to three independent equations: (i) $\mu_s(0^-) = \mu_s(0^+)$, (ii) $\mu_s(0^-) = \mu_s(0^+)$, and (iii) $j_s(0^-) = j_s(0^+)$. The current can be calculated using $j_f = 0$ for $x < 0$ and $j_f = 0$ for $x > 0$.

The spin injection in the semiconductor is usually defined via the spin polarization of the current $\alpha(x) = (j_s(x) - j_s(0^+))/J$, which is proportional to the spin polarization of the electron density $n_s$, $n_d$.

Thus the solution of $n_s = n_d$ in Eq. (8) indicates $\alpha(x) = \alpha_0 e^{-x L_d}$, where $\alpha_0$ is the spin injection efficiency. We obtain an equation for $\alpha_0$, noting 1 = $-k_B T/e E L_d$.

\[ 2L^{(f)}(\alpha - \sigma_f) = \frac{k_B T}{e E} \ln \left( -k_B T/e E L_d + \alpha_0 \right), \]

where $\alpha_s = \sigma_s^2 + \sigma_s^4$, and $p_f = (\sigma_s^4 - \sigma_s^2)/\sigma_s$ is the spin polarization in the metal. We solve Eq. (11) and plot the spin injection efficiency $\alpha_0$ as a function of the electric field in Fig. 2. We see that the electric field can enhance the spin injection efficiency considerably. When $\Delta n_s / n_0 \ll 1$, i.e., for small spin polarization in the semiconductor, $\alpha_0$ can be expressed in a compact form,

\[ \alpha(x) = \frac{L^{(f)}}{1 - p_f^2 \sigma_f} \left( \frac{L^{(f)}}{1 - p_f^2 \sigma_f} \right)^{-1} \frac{C_L^{(f)}}{(1 - p_f^2 \sigma_f) e^{-x L_d}}. \]

This remarkable expression shows that the electric-field effects on spin injection can be described in terms of the two field-induced diffusion lengths. Both diffusion lengths affect spin injection favorably but in a different manner. The upstream length $L_u$ controls the relevant resistance in the semiconductor, which determines the spin injection efficiency. With increasing field this effective resistance, $L_u/\sigma_s$, becomes smaller, and accordingly the spin injection efficiency is enhanced. As $E \rightarrow \infty$, $\alpha_0$ goes to $p_f$. The transport distance of the injected spin polarization in the semiconductor, however, is controlled by the downstream length $L_d$. As the field increases, this distance becomes longer.

We now contrast Eq. (12) with that obtained by previous calculations\(^2,^4\) based on Eq. (1). The spin injection

\[ \alpha(x) = \frac{L^{(f)}}{1 - p_f^2 \sigma_f} \left( \frac{L^{(f)}}{1 - p_f^2 \sigma_f} \right)^{-1} \frac{C_L^{(f)}}{(1 - p_f^2 \sigma_f) e^{-x L_d}}. \]

is given by the zero-field result of Eq. (12). As $L^{(f)} \ll L^{(s)}$ and $\sigma_s \gg \sigma_f$, the effective resistance in the metal, $L^{(f)}/\sigma_s$, is much less than its counterpart in the semiconductor, $L^{(s)}/\sigma_f$. Thus Eq. (13) suggests that this resistance mismatch makes it virtually impossible to realize an appreciable spin injection.
from a ferromagnetic metal to a semiconductor. However, our more general description of the spin transport in semiconductors indicates that the effective semiconductor resistance to be compared with \( L^{(f)} / \sigma_f \) should be \( L_u / \sigma_f \) rather than \( L^{(s)} / \sigma_s \). Since \( L_u \) can be smaller than \( L^{(s)} \) by orders of magnitude in the high-field regime, this “conductivity mismatch” obstacle may be overcome with the help of strong electric fields, or equivalently, large injection currents.\(^\text{16}\) For example, if we choose the following parameters of a spin injection device, \( p_f = 0.5 \) and \( L^{(f)} = 60 \text{ nm} \) [as in Co (Ref. 17)], \( L^{(s)} = 2 \text{ \mu m} \) [as in GaAs (Ref. 6)], for Co/GaAs structures, \( \sigma_f \approx 10^4 \sigma_s \), the spin injection efficiency increases from \( 2 \times 10^{-6} \) at zero field to \( 2\% \) at \( |E| / k_B T = 5 \text{ nm}^{-1} \). For a ferromagnetic semiconductor with \( p_f = 0.5 \) and \( \sigma_f \approx 100 \sigma_s \), the spin injection efficiency increases from 0.02% at zero field to \( 2\% \) at \( |E| / k_B T = 0.05 \text{ nm}^{-1} \), which corresponds to \( |E| = 125 \text{ V/cm} \), or \( |J| = 1250 \text{ A/cm}^2 \) for a semiconductor conductivity \( \sigma_s = 10 \text{ (\Omega cm)}^{-1} \), at \( T = 3 \text{ K} \).

For structures with a spin-selective interfacial barrier between the ferromagnetic metal and the semiconductor, which have been suggested to increase spin injection in the low-field regime,\(^\text{5,6}\) we find

\[
\alpha(x) = \left[ \frac{L^{(f)}}{(1-p_f^2)\sigma_f} + \frac{L_u}{\sigma_s} + \frac{G_1 + G_1'}{4G_i G_i'} \right]^{-1} \\
\times \left[ \frac{p_f L^{(f)}}{(1-p_f^2)\sigma_f} + \frac{G_1 - G_1'}{4G_i G_i'} \right] e^{-x/L_d}, \tag{14}
\]

where \( G_{1(1)} \) is the interfacial conductance for up-spin (down-spin) electrons. This expression shows that the electric field and the spin-selective interfacial resistance both enhance spin injection, but in independent ways. The spin injection enhancement from a spin-selective barrier would therefore become even more pronounced in the high-field regime. For a barrier resistance dominating over the ferromagnet’s effective resistance, and \( G_1 = 2G_1' \), the maximum spin injection achievable is 33%. For a Co/GaAs structure with \( G_1 = 2G_1' = 6 \times 10^5 \text{ (\Omega cm)}^{-2} \), the spin injection efficiency increases from 2% at zero field to 10% with \( |E| = 840 \text{ V/cm} \) at \( T = 300 \text{ K} \), or \( |E| = 8.4 \text{ V/cm} \) at \( T = 3 \text{ K} \). The two mechanisms have distinct electric-field and temperature dependences: field-enhanced spin injection increases with the electric field (current) and decreases with the temperature; whereas interface-enhanced spin injection depends on neither the electric field nor the temperature. This difference may help clarify the origin of spin injection enhancement in different structures. Recently Fert and Jaffrès have identified the relative values of \( L^{(s)} / \sigma_s \) to barrier resistance as governing spin injection.\(^\text{18}\) Equation (14) shows instead it is \( L_u / \sigma_s \) that plays this role. As a result, a less resistive barrier is required to generate a particular amount of spin-polarized current, and the power dissipation of the circuit can be much less. The combination of spin-selective barrier and electric-field enhancement may help to understand the large spin injection percentages from ZnMnSe to ZnSe,\(^\text{6,9}\) from GaMnAs to GaAs,\(^\text{10}\) and from Fe to GaAs,\(^\text{12,19,20}\) as well as the dramatic increase in spin injection with current in Ref. 12.

In summary, we have derived the drift-diffusion equation for spin polarization in a semiconductor by consistently taking into account electric-field effects and nondegenerate electron statistics. This equation provides a framework to understand spin transport in semiconductors. We have identified a high-field diffusive regime which has no analog in metals. In this regime, there are two distinct spin-diffusion lengths, i.e., the upstream and downstream spin-diffusion lengths. The high-field description of the spin transport in semiconductors predicts that the electric field can effectively enhance spin injection from a ferromagnetic metal into a semiconductor and substantially increase the transport distance of the spin polarization in semiconductors. Our results suggest that the “conductivity mismatch” obstacle in spin injection may be overcome with the help of high-field injection in the diffusive regime.

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1 See, e.g., S.A. Wolf et al., Science 294, 1488 (2001), and references therein.
16 Although it has been realized that spin injection can be enhanced by increasing the total injection current (Ref. 4), the electric-field effects on spin transport were not taken into account and therefore the physics of the high-field regime was not captured.